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ISOLATING THE DECAY RATE OF COSMOLOGICAL GRAVITATIONAL POTENTIAL

Pengjie Zhang
Shanghai Astronomical Observatory, Chinese Academy of Science, 80 Nandan Road, Shanghai, China, 200030 and
Joint Institute for Galaxy and Cosmology (JOINGC) of SHAO and USTC

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ABSTRACT

The decay rate of cosmological gravitational potential measures the deviation from Einstein-de Sitter universe and can put strong constraints on the nature of dark energy and gravity. Usual method to measure this decay rate is through the integrated Sachs-Wolfe (ISW) effect-large scale structure (LSS) cross correlation. However, the interpretation of the measured correlation signal is complicated by the galaxy bias and matter power spectrum. This could bias and/or degrade its constraints to the nature of dark energy and gravity. But, combining the lensing-LSS cross correlation measurements, the decay rate of gravitational potential can be isolated. For any given narrow redshift bin of LSS, the ratio of the two cross correlations directly measures $[d\ln D_\phi/d\ln a]H(z)/W(\chi,\chi_s)$, where D_ϕ is the linear growth factor of the gravitational potential, H is the Hubble constant at redshift z, $W(\chi,\chi_s)$ is the lensing kernel and χ and χ_s are the comoving angular diameter distance to lens and source, respectively. This method is optimal in the sense that (1) the measured quantity is essentially free of systematic errors and is only limited by cosmic variance and (2) the measured quantity only depends on several cosmological parameters and can be predicted from first principles unambiguously. Though fundamentally limited by inevitably large cosmic variance associated with the ISW measurements, it can still put useful independent constraints on the amount of dark energy and its equation of state. It can also provide a powerful test of modified gravity and can distinguish the Dvali-Gabadadze-Porrati model from ΛCDM at $>2.5\sigma$ confidence level.

Subject headings: Cosmology: the large scale structure: cosmic microwave background: gravitational lensing

1. INTRODUCTION

The decay rate of gravitational potential is a powerful probe of dark energy (Crittenden & Turok 1996) and the nature of gravity (Lue et al. 2004a,b; Zhang 2005). Its direct observational consequence is the integrated Sachs-Wolfe (ISW) effect (Sachs & Wolfe 1967), one class of secondary CMB temperature fluctuations. However, since the signal is overwhelmed by primary cosmic microwave background (CMB), it has to be measured by cross correlating the large scale structure (LSS) (Crittenden & Turok 1996; Seljak & Zaldarriaga 1999b), or from CMB polarization induced by intra-cluster electron scattering (Cooray et al. 2004). Progresses in precision CMB measurements and large scale galaxy surveys have enabled detections of the ISW-LSS cross correlation and confirmed the decay of cosmological gravitational potential (Fosalba et al. 2003; Scranton et al. 2003; Afshordi et al. 2004; Boughn & Crittenden 2004; Fosalba & Gaztanaga 2004; Nolta et al. 2004; Vielva et al. 2004; Padmanabhan 2005). Along with the flatness of the universe constrained from the CMB (Spergel et al. 2003), detections of the gravitational potential decay have already provided strong evidence of the existence of dark energy (Corasaniti et al. 2005) and constraints to the dark energy equation of state can be further improved by future observations (Pogosian et al. 2005). However, the strength of the cross correlation signal depends not only on the dark energy density $\Omega_{\rm DE}$ and its equation of state parameter $w_{\rm DE}$, but also on the matter power spectrum and evolving galaxy bias[1], which can not be predicted from first principles. These nuisance parameters and modeling uncertainties could degrade the power of ISW-LSS cross correlation to constrain dark energy. However, as proposed in this paper, by the aid of gravitational lensing-LSS cross correlation, the evolution of gravitational potential can be isolated.

The ISW effect and lensing-LSS probe the same 3D gravitational potential ϕ, with different prefactors. Since galaxy redshifts are observable, one can correlate ISW or lensing with galaxies in a narrow redshift bin. For such narrow redshift bin, the ISW-galaxies cross correlation measures $\dot\phi$-δ_g cross correlation, while the lensing-galaxies cross correlation measures ϕ-δ_g cross correlation. Here, δ_g is the over-density of galaxies. The ratio of the two correlations in the same redshift bin at the same scale then measures $\dot\phi/\phi$, with prefactor which only depends on the geometry of the Universe, but not galaxy bias nor matter power spectrum. Such measurement involves essentially no assumptions and least amount of unknown cosmological parameters, so it can put robust constraints on cosmology and has the power to distinguish dark energy from modified gravity. Furthermore, since these two cross correlation measurements are correlated, errors in the denominator and numerator partly cancel. The S/N of the measured ratio is slightly better

Electronic address: pjzhang@shao.ac.cn

[1] The cross correlation signal is proportional to r, the cross correlation coefficient between galaxy overdensity and the gravitational potential. In parameter fitting combining galaxy power spectrum measurement, $r=1$ at relevant scales is implicitly assumed. But stochasticity could cause r to deviate from unity. So in principle, r should also be treated as a free parameter to be marginalized.

than the ISW-LSS measurement. Since $\dot{\phi}/\phi$ of each redshift can be recovered, one obtains stronger cosmological constraints than that of the ISW-lensing measurement (Seljak & Zaldarriaga 1999b).

2. ISOLATING THE DECAY RATE OF GRAVITATIONAL POTENTIAL

Time variation in the cosmological gravitational potential causes temperature fluctuations in CMB (Sachs & Wolfe 1967)

$$\frac{\Delta T}{T_{\rm CMB}} = \int [\dot{\phi} - \dot{\psi}] a d\chi \ , \tag{1}$$

where ϕ and ψ are two gravitational potentials in the Newtonian gauge. In dark energy models, at late time, there is no anisotropic stress and $\phi = -\psi$. The ISW-LSS cross correlation power spectrum is given by (Peebles 1973)

$$C_{Ig} = 4\pi \int \Delta^2_{\phi\delta}(k, z=0)\frac{dk}{k} A_I(k,l) A_g(k,l) \ . \tag{2}$$

Here, galaxies at $\bar{z} - \Delta z/2 < z < \bar{z} + \Delta z/2$ have been adopted to be tracers of the LSS. $A_g = \int_{\chi_1}^{\chi_2} j_l(k\chi) n(\chi) D_g d\chi$ and $A_I \equiv 2\int_0^{\chi_{\rm CMB}} j_l(k\chi)\dot{D}_\phi a d\chi$, where $\chi_1, \chi_2, \chi_{\rm CMB}$ are the comoving angular diameter distance to $\bar{z} - \Delta z/2$, $\bar{z} + \Delta z/2$ and the last scattering surface, respectively. D_g and D_ϕ are the linear growth factors of galaxy over-density and gravitational potential ϕ, respectively. $n_g(\chi)$ is the number of galaxies per distance interval. $\Delta^2_{\phi\delta}$ is the cross correlation power spectrum variance between ϕ and galaxy over-density δ. j_l is the spherical Bessel function. This equation assumes linear evolution in ϕ and galaxy over-density, which should be valid at large scales where almost all ISW signal comes from. But it does not assume the galaxy bias to be scale independent. Eq. 2 and 3 (please refer to the appendix for the derivation) do not require the small angle approximation and the Limber's approximation and thus apply to all angular scales and redshift bins relevant.

One can construct the projected gravitational potential Φ (or its harmonic mode) from CMB lensing (Seljak & Zaldarriaga 1999a; Zaldarriaga & Seljak 1999; Hu & Okamoto 2002), 21cm background lensing (Zahn & Zaldarriaga 2005), cosmic shear (see Refregier (2003) for a recent review) and cosmic magnification (Zhang & Pen 2005). $\Phi(\hat{n}) = -\int d\chi(\phi - \psi) W(\chi, \chi_s)$, where $W = (1 - \chi/\chi_s)/\chi$, χ and χ_s are the comoving angular diameter distance to lens and source. The cross correlation between Φ and LSS is

$$C_{\Phi g} = 4\pi \int \Delta^2_{\phi\delta}(k, z=0)\frac{dk}{k} A_\Phi(k,l) A_g(k,l) \tag{3}$$

where $A_\Phi = 2\int_0^{\chi_s} j_l(k\chi) W(\chi,\chi_s) D_\phi d\chi$, when $\phi + \psi = 0$.

Eq. 2 & 3 can be further simplified by the Limber's approximation (Limber 1954; Kaiser 1998). Afshordi et al. (2004) showed that, even for a shallow survey such as 2MASS, which has an equivalent $\Delta z \sim 0.1$, the Limber's approximation is still valid to several percent level for $l \geq 3$. We then adopt $\Delta z = 0.2$ and apply the Limber's approximation. Under this approximation, we have

$$C_{Ig} = \frac{4\pi^2}{l^3}\int_{\chi_1}^{\chi_2} \Delta^2_{\phi\delta}(\frac{l}{\chi}, z=0) n_g D_g \dot{D}_\phi a \chi d\chi \tag{4}$$

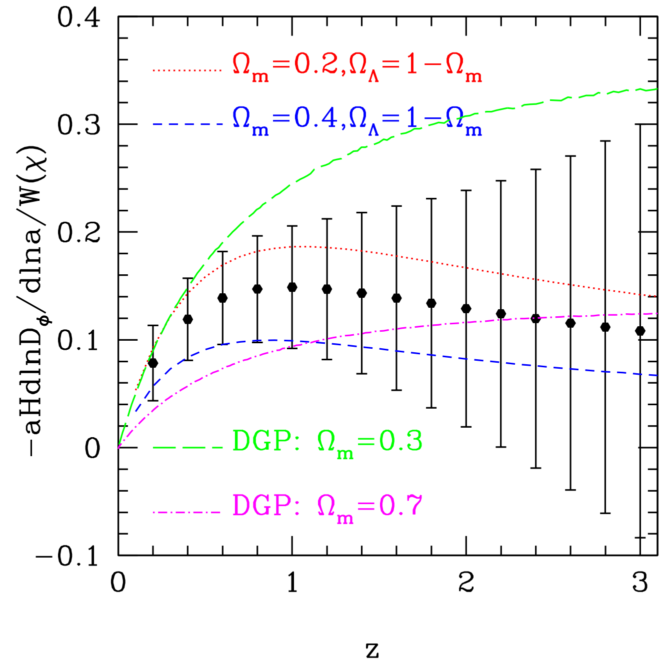

FIG. 1.— The accuracy of measured $f \equiv aH d\ln D_\phi/d\ln a/W(\chi)$, assuming both CMB and galaxy surveys cover the full sky. Fiducial cosmology has $(\Omega_m, \Omega_\Lambda, \Omega_b, h, \sigma_8) = (0.268, 0.732, 0.044, 0.71, 0.84)$. Transfer function is calculated using the fitting formula of (Eisenstein & Hu 1998).

Since D_ϕ, $\dot{D}_\phi$, a, χ, $\Delta^2_{\phi\delta}(\frac{l}{\chi})$ do not vary significantly over the redshift range $[\bar{z} - \Delta z/2, \bar{z} + \Delta z/2]$ as long as that $\bar{z} \gtrsim \Delta z$ and n_g does not vary significantly, we have

$$C_{Ig} \simeq \frac{4\pi^2}{l^3}[\Delta^2_{\phi\delta}(\frac{l}{\chi}, z=0) n_g D_g \dot{D}_\phi a\chi]|_{\bar{z}}(\chi_2 - \chi_1) \ . \tag{5}$$

For the same reason, we have

$$C_{\Phi g} \simeq \frac{4\pi^2}{l^3}[\Delta^2_{\phi\delta}(\frac{l}{\chi}, z=0) n_g D_g D_\phi W\chi]|_{\bar{z}}(\chi_2 - \chi_1) \ . \tag{6}$$

These approximations are accurate to several percent level for relevant l and $\bar{z} \geq 0.2$. We then have

$$C_{Ig} \simeq f(\bar{z}) C_{\Phi g} \ . \tag{7}$$

Here, $f(z) = [(d\ln D_\phi/d\ln a)]aH/W(\chi)$, is the key quantity we want to measure. Given the large cosmic variance in the C_{Ig} measurement, one can safely neglect any possible errors caused by this approximation. Since $C_{Ig} \simeq f(\bar{z})C_{\Phi g}$ is a key relation in this paper, here we show another proof. The Limber's approximation is achieved by taking the limit that $j_l(x) \rightarrow \sqrt{\pi/(2l+1)}\delta_D(l + 1/2 - x)$. Under this limit, for each l, only those k between $[(l+1/2)/\chi_2, (l+1/2)/\chi_1]$ contribute to C_{Ig}, since only those k have non-vanishing $A_g(k,l)$. For these k, $\int_0^{\chi_1} j_l(k\chi)\dot{D}_\phi a d\chi \simeq 0$, $\int_{\chi_2}^{\chi_{\rm CMB}} j_l(k\chi)\dot{D}_\phi a d\chi \simeq 0$ and $A_I(k,l) \simeq \int_{\chi_1}^{\chi_2} j_l(k\chi)\dot{D}_\phi a d\chi$. Similarly, we have $A_\Phi \simeq 2\int_{\chi_1}^{\chi_2} j_l(k\chi) W(\chi,\chi_s) D_\phi d\chi$ for these k. Thus, we have $C_{Ig} \simeq f(\bar{z})C_{\Phi g}$, from Eq. 2 & 3.

$f(z)$ is determined by the matter density, dark energy density and equation of state, but it does not depend on galaxy bias or matter power spectrum. Since H_0,

the Hubble constant at present, in $H(z)$ and $W(\chi)$ cancel each other, f does not depend on H_0, too. $f(z)$ of each redshift bins can be estimated from the estimator $\hat{f} = \sum_l C_{Ig}(l) w_l / \sum C_{\Phi g} w_l$, where w_l is the weighting function. For Φ reconstructed from future cosmic shear, cosmic magnification and CMB lensing, the fractional error in the denominator is small and one can do Taylor expansion to estimate the error in $\hat{f}$. For this limit, we have

$$\frac{\Delta f^2}{f^2} = \frac{\sum_l \Delta C_{Ig}^2 w_l^2}{[\sum_l C_{Ig} w_l]^2} + \frac{\sum_l \Delta C_{\Phi g}^2 w_l^2}{[\sum_l C_{\Phi g} w_l]^2} - \frac{2\sum_l \langle \Delta C_{Ig} \Delta_{\Phi g}\rangle w_l^2}{\sum_l C_{Ig} w_l \sum_l C_{\Phi g} w_l} . \tag{8}$$

Here,

$$\Delta C_{Ig}^2(l) = \frac{\tilde{C}_{\rm CMB}\tilde{C}_g + C_{Ig}^2}{(2l+1)f_{\rm sky}} ,$$
$$\Delta C_{\Phi g}^2(l) = \frac{\tilde{C}_{\Phi}\tilde{C}_g + C_{\Phi g}^2}{(2l+1)f_{\rm sky}} , \tag{9}$$
$$\langle \Delta C_{Ig} \Delta C_{\Phi g}\rangle = \frac{C_{\Phi I}\tilde{C}_g + C_{Ig}C_{\Phi g}}{(2l+1)f_{\rm sky}}$$

are the statistical errors of C_{Ig}, $C_{\Phi g}$ and cross correlation between them, respectively. $\tilde{C}_{\rm CMB}$, $\tilde{C}_g$ and $\tilde{C}_\Phi$ are the sums of corresponding signals and associated contaminations. For Φ reconstructed from CMB lensing of a very low noise CMB experiment, $\tilde{C}_\Phi \simeq C_\Phi$ at $l < 200$ (Hu & Okamoto 2002). For Φ reconstructed from the cosmic magnification and cosmic shear of future radio and optical surveys, this holds over even larger l range. Since most ISW-LSS cross correlation signal comes from $l \lesssim 100$ (Afshordi 2004) and 21cm emitting galaxies have high surface density, we neglect shot noise term[2] and approximate $\tilde{C}_g = C_g$. Since $C^{\rm CMB}/C^{\rm ISW} \gg 1$, errors in C_{Ig} will dominate over errors in $C_{\phi g}$ and cross correlations. Furthermore, the last two terms in Eq. 8 partly cancel. So it is a good approximation to neglect the last two terms. Under this simplification, we obtain the minimum variance estimator $w_l = \frac{C_{Ig}(l)}{\Delta C_{Ig}^2(l)}$ and the minimum variance is

$$\frac{\Delta f^2}{f^2} \simeq \left(\sum_l \frac{C_{Ig}^2}{\Delta C_{Ig}^2}\right)^{-1} \simeq \left(\sum_l \frac{(2l+1) f_{\rm sky} C^{\rm ISW}}{r^2 C^{\rm CMB}}\right)^{-1} \tag{10}$$

where $C^{\rm ISW}$ is the ISW power spectrum of the corresponding redshift bin and r is the cross correlation coefficient between the ISW effect of the corresponding redshift bin and galaxies. For narrow redshift bins we choose, r is very close to unity.

We choose the fiducial cosmology with best fit WMAP parameters $\Omega_m = 0.268$, $\Omega_\Lambda = 1 - \Omega_m$, $\Omega_b = 0.044$,

[2] For the estimations of LSS clustering signal and shot noise, biggest uncertainties are (1) HI (*neutral hydrogen*) mass function at high z, (2) 21cm emitting galaxy bias and (3) specifications of 21cm experiments. If one adopts HI mass functions calibrated against observations of damped Lyman-α systems and Lyman limit systems, SKA can detect $\sim 10^9$ galaxies at $z \sim 3$ in five years across the whole sky, for a field of view 10 deg^2 at ~ 300 Mhz (for details of the calculation, see, e.g. Zhang & Pen (2006)). For SKA, detection threshold of HI mass at $z = 3$ is $\sim 10^9 M_\odot$, so detected galaxies are likely having biases bigger than one. Then, one can neglect the shot noise term with respect to C_g

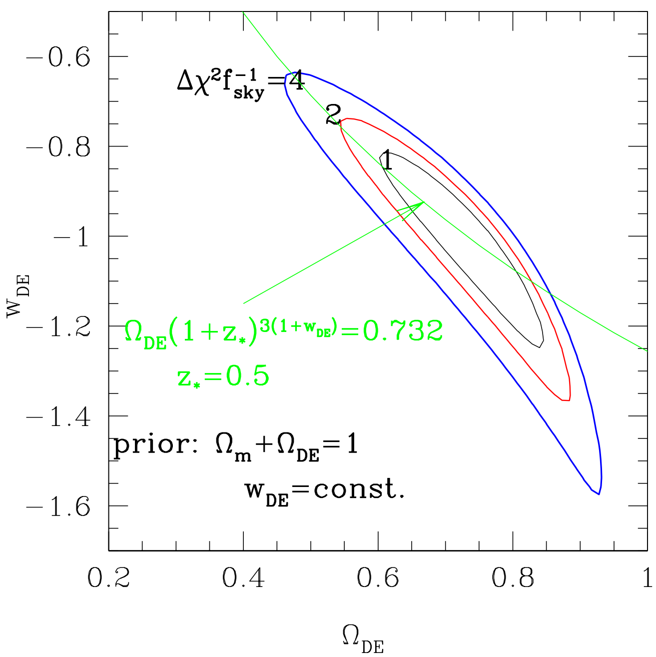

FIG. 2.— $\Omega_{\rm DE}$-$w_{\rm DE}$ contour. Since most signals come from $z_* \sim 0.5$, $d\ln D_\phi/d\ln a$ at $z_* \sim 0.5$ should be close to the fiducial cosmology in order to get a good fit. This requires that the dark energy density $\Omega_{\rm DE}(1+z_*)^{3(1+w_{\rm DE})} \simeq \Omega_\Lambda^{\rm fiducial}$. We plot the line with $z_* = 0.5$.

$h = 0.71$, $\sigma_8 = 0.84$ and primordial power index $n = 1$. Transfer function is calculated according to Eisenstein & Hu (1998). We use this fiducial cosmology to estimate the S/N of f to be measured from future observations. We then use this fiducial S/N to forecast dark energy and gravity constraints. We choose 21cm emitting galaxies as tracers of LSS. 21cm emission is less affected by dust and 21cm surveys such as SKA can cover the whole sky. They also have the advantage to go deep into $z > 3$ and allow the measurement of f at $z \sim 3$. We choose the CMB reference experiment discussed in Hu & Okamoto (2002) to produce a full sky lensing map. Galaxy bin size is chosen to be $\Delta z = 0.2$. For each galaxy redshift bin, one obtains a measure of $f(z)$ at that redshift. The result is shown in Fig. 1. For the fiducial model, the decay of ϕ can be detected at $z < 2$. Since the physical separation of two redshift bins are much larger than the galaxy correlation length, LSS of any two redshift bins are uncorrelated, f of each bins are close to be independent[3]. We will do a reduced χ^2 analysis to estimate the dark energy constraints and gravity constraints. The reduced χ^2 is defined as

$$\Delta\chi^2 = \sum_i \frac{(f(z_i) - f_i^{\rm fid})^2}{\Delta f_i^{\rm fid,2}} , \tag{11}$$

where $f(z_i)$ and $f^{\rm fid}$ are the predicted f of a given dark

[3] Because the ISW effect is projected over the redshift range from zero to ~ 1100, there does exist a correlation between $C_{Ig}(l)$ of different redshift bins. The same is true for $C_{\Phi g}$. However, statistical errors of f in each redshift bins are dominated by primary CMB. So the cross correlation coefficients between error bars of different redshift bins are close to zero. It is safe to neglect such correlations and take f measurements of each redshift bins as uncorrelated.

energy model or gravity model and of the fiducial model, respectively.

3. CONSTRAINING DARK ENERGY

The quantity f is very sensitive to the amount of dark energy. To visualize it, we plot those of $\Omega_m = 0.2, 0.4$ flat ΛCDM in Fig. 1. One can see that these two cosmologies can be distinguished from the fiducial cosmology at $> 3\sigma$ confidence level (C.L.). Given a flat ΛCDM prior, Ω_Λ can be constrained to $0.73^{+0.05}_{-0.07}$ at 2σ C.L..

We want to constrain both $\Omega_{\rm DE}$ and $w_{\rm DE}$ simultaneously. At super-horizon and near horizon scales, dark energy fluctuation can contribute $\sim 10\%$ to the decay of ϕ (Bean & Dore 2004; Hu & Scranton 2004). To calculate the dark energy fluctuation, one extra parameter, the sound speed c_s, is required. Fortunately, the ISW signal (weighted by noise) mainly comes from sub-horizon scale. For the purpose of this paper, we can neglect the dark energy fluctuation in the parameter estimation. With this simplification, D_ϕ is given by

$$D^{''}_\phi + D^{'}_\phi\left(\frac{5}{a}+\frac{H^{'}}{H}\right)+\frac{D_\phi}{a^2}\left(3+\frac{H^{'}a}{H}-\frac{3}{2}\frac{\Omega_m H_0^2}{a^3H^2}\right)=0\ , \quad (12)$$

where $^{'} \equiv d/da$ and $^{''} = d^2/da^2$. We numerically solve this equation with the initial condition that when $a \to 0$, $D_\phi \to$const. and $D^{'}_\phi \to 0$. The results is shown in Fig. 2. The constraints are not very impressive, comparing to other methods. But since the theoretical prediction only involves general relativity and parameter fitting involves the least amount of free parameters, the constraints are less affected by model uncertainties and the existence of dark energy can be confirmed at high confidence level.

4. DISTINGUISHING DARK ENERGY FROM MODIFIED GRAVITY

Precision cosmology provides crucial tests of general relativity at cosmological scales. Modifications to general relativity change not only the expansion history of the Universe, but also the LSS. For modified gravities which reproduce the expansion history of dark energy models (e.g. Dvali et al. (2000); Carroll et al. (2004, 2005); Mena et al. (2005)), the structure growth is in general different. Thus, the measured f from our methods can provide crucial tests for such models. In general, modifications to general relativity causes D_ϕ to be scale dependent. In this case, one has to choose much narrower l bin to isolate $f(z)$ of each scale. Since the relative error in the denominator can approach unity for narrow l bin, more sophisticated estimator is required to isolate f. For simplicity, in this paper we only discuss the Dvali-Gabadadze-Porrati (DGP) model (Dvali et al. 2000), which preserves the scale independent $D_{\phi-\psi}$ (Lue et al. 2004b; Koyama & Maartens 2005).

For a flat DGP model, the expansion history is given by $H^2 = \frac{H}{r_0} + H_0^2\Omega_m a^{-3}$, where $r_0 = 1/[H_0(1-\Omega_m)]$. The two Newtonian potential ϕ and ψ no longer follow the relation $\phi+\psi = 0$. The evolution of $\phi-\psi$ is governed by

$$D^{''}_{\phi-\psi}+D^{'}_{\phi-\psi}\left(\frac{5}{a}+\frac{H^{'}}{H}\right)+\frac{D_{\phi-\psi}}{a^2}\left(3+\frac{H^{'}a}{H}-\frac{3}{2}\frac{\Omega_m H_0^2}{a^3H^2}\frac{G_{\rm eff}}{G}\right)=0 \quad (13)$$

where $G_{\rm eff}/G = 1+1/3\beta$ and $\beta = 1+2r_0H^2/(1-2r_0H) < 0$ (Lue et al. 2004b; Koyama & Maartens 2005). This

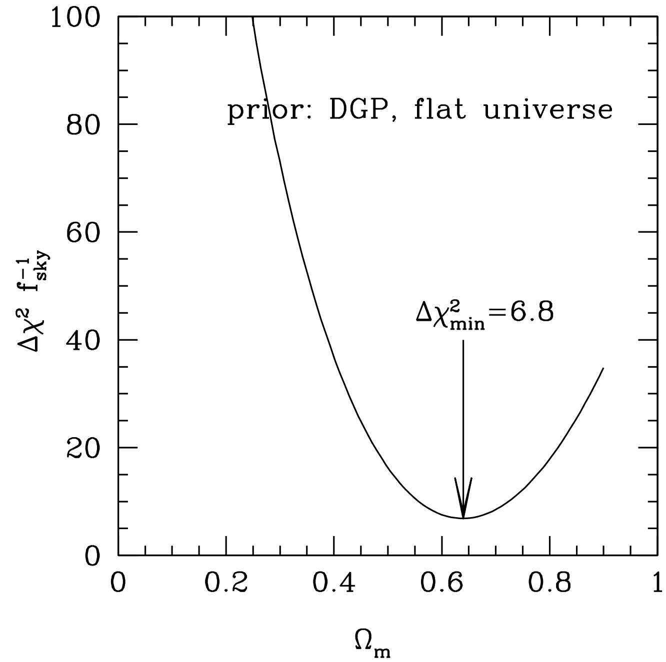

FIG. 3.— Distinguishing DGP from the fiducial ΛCDM cosmology. Since at high redshifts gravitational potential decays much faster in DGP than in ΛCDM cosmology, DGP can be distinguished from ΛCDM at $> 2.5\sigma$ confidence level.

equation holds at sub-horizon scales with $k \gg aH$ and $k \gg 1/r_0$. At $z \gtrsim 2$, $l < 10$ modes do not satisfy these conditions. But at these redshifts, most signal comes from $l > 10$ and throwing away $l < 10$ modes does not degrade the f measurement significantly. For simplicity, we neglect this complexity.

We find that flat DGP model can be distinguished from the fiducial ΛCDM cosmology at $> 2.5\sigma$ C.L.. Fig. 1 demonstrates this point. When $a \to 0$, the expansion history of a DGP model resembles a $w_{\rm DE} = -1/2$ dark energy model (or ΛCDM with curvature). But gravitational potential in DGP decays faster, due to extra suppression caused by smaller effective Newton's constant $G_{\rm eff}$. Comparing to the fiducial ΛCDM model, a low Ω_m DGP model causes too fast gravitational potential decay at high redshifts while a high Ω_m DGP model causes too slow gravitational potential decay at low redshifts. So it is hard for DGP model to reproduce f and its redshift dependence in the fiducial ΛCDM cosmology. One can infer similar conclusion From Fig. 3, where $w_{\rm DE} > -0.6$ dark energy model is excluded at $> 90\%$ confidence level. Since gravitational potential in DGP decays faster than a $w_{\rm DE} = -1/2$ dark energy model, DGP should be distinguished from the fiducial model with higher C.L.. This can be better understood from the asymptotic behavior of $\dot{D}_\phi$. When $a \to 0$, $d\ln D_{\phi-\psi}/d\ln a \to -[11/16][(1-\Omega_m)/\Omega_m^{1/2}]a^{3/2}$ for DGP and $d\ln D_{\phi-\psi}/d\ln a \to -[3(1-w_{\rm DE})/(6-5w_{\rm DE})][(1-\Omega_m)/\Omega_m]a^{-3w_{\rm DE}}$ for dark energy. At high redshifts, gravitational potential decays faster in DGP than in dark energy models with $w_{\rm DE} \le -1/2$ and much faster than in ΛCDM cosmology. It is this feature that allows a clear discrimination between DGP and many

dark energy models using the ISW effect.

5. DISCUSSION

We have shown how to isolate the decay rate of gravitational potential from ISW-LSS and lensing-LSS cross correlation measurements. The measured decay rate, with prefactors only depend on geometry of the universe can put robust constraints on dark energy and the nature of gravity, free of many theoretical uncertainties. The accuracy of such measurement is dominated by statistical fluctuations dominated by primacy CMB. This allows us to safely neglect several possible systematics. (1) Dark energy fluctuations. As shown in Bean & Dore (2004), such fluctuations are at most 10% those of dark matter fluctuation. For sound speed close to unity, these fluctuations effectively vanishes at most scales we are interested and the overall effect can be neglected. (2) Time evolution of $f(z)$. Since the bin size Δz is not infinitely small, the simplification $f(z) \simeq f(\bar{z})$ for $\bar{z} - \Delta z/2 < z < \bar{z} + \Delta z/2$ causes an relative error $\simeq [d^2f/dz^2/f][(\Delta z)^2/12] \ll 1$, for adopted $\Delta z = 0.2$. (3) Correlated error bars in f measurement.

We keep in caution that foregrounds of CMB, LSS and lensing may be correlated and their effects may become non-negligible toward the galactic plane. In this case, one needs to mask the galactic plane, at the expense of losing statistical accuracy, scaled as $f_{\rm sky}$ (fractional sky coverage).

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